MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 16, September 10, 2013

1. Let n be a positive integer such that $100n + 64$ and $201n + 64$ are both 4-digit square numbers. Find n.

2. Show that any straight line passing through the centre of a parallelogram (i.e. the intersection of the diagonals) divides the parallelogram into two equal areas.

3. (a) Show that any convex polygon of area 1 can be enclosed in a parallelogram of area 2.

   (b) Show that a triangle of area 1 cannot be enclosed in a parallelogram of area less than 2.

4. (a) What are the possible remainders when we divide a square integer by 4?

   (b) Suppose we have three numbers such that the sum of any two different ones is a square. Show that at most one of the numbers can be odd.

   (c) Can you find such an example with exactly one of the number being odd?

5. Shown below (Figure 1) is a tourist map of 18th century Königsberg, provided to us by Euler Tourism showing the Pregel River and the seven bridges which cross it. These bridges are particularly stunning examples of Prussian architecture and Euler Tourism highly recommends taking a walk across each bridge. However, We are lazy and want to find a route which crosses each bridge exactly once (we don't mind if the walk starts and ends in different locations).

   (a) Show that, except for the beginning or end of our walk, each land mass must have an even number of bridges leading to neighbouring land masses for this route to be possible.

   (b) Deduce that Königsberg does not permit such a lazy walk.

   (c) What's the smallest number of bridges that must be built to accomdate lazy tourists such as ourselves?

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[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni. Senior Problems by David Treeby
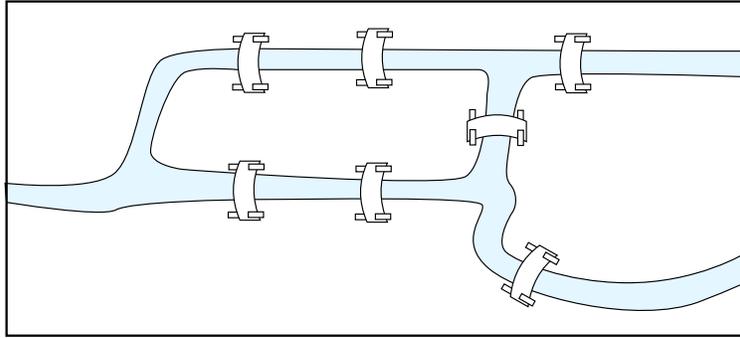
Figure 1: 18th Century Königsberg tourist map, Euler Tourism

6. What is the maximum number of Knights that can be placed on an 8×8 chessboard so that no two Knights attack each other. (Bonus points for the maximum number of Knights such that each Knight attacks exactly one other Knight).

Senior Questions

1. (a) A palindromic number is one which is the same read forwards as backwards, e.g. 101. How many positive palindromic numbers are there less than 10^{100}.

2. Compute the sum
$$1 + 11 + 111 + \cdots + \underbrace{111111 \cdots 1}_{2013 \text{ 1's}}.$$