

MATHEMATICS ENRICHMENT CLUB.¹
Problem Sheet 17, September 17, 2013

1. How many integers between 100 and 999 have distinct odd digits?
2. A palindromic number is one that reads the same from left-to-right as from right-to-left. The number 121 is a palindromic square. Can you find others with 3, 4 or 5 digits?
3. Find the side length of the square whose base lies on the base of an isosceles triangle with sides 10, 10 and 12, with two vertices touching the equal sides of the triangle.
4. Find the largest positive integer that when divided into each of 364, 414 and 539 leave the same remainder.
5. (a) The sequence a_n is defined by $a_1 = 0$ and $a_n = n + a_{n-1}$. Find the value of a_6 .
(b) Find a formula for a_n .
(c) The sequence b_n is defined by $b_1 = 0$ and $b_n = n^2 + b_{n-1}$. Find the value of b_6 .
(d) Find a formula for b_n .
6. Given a triangle ABC , draw a straight line through each vertex parallel to the opposite side, thereby forming a new triangle $A_1B_1C_1$ with A_1 opposite A and so on.
(a) Show that the altitude drawn from A in the triangle ABC is the perpendicular bisector of B_1C_1 . (Hint: Look for parallelograms.)
(b) Conclude that the three altitudes of a triangle are concurrent.
7. Let AB be a chord of a circle centre O and let P be a point on its circumference. If $\angle APB = \angle AOB$, find this angle.

Senior Questions

1. What is the coefficient of x^{17} and the coefficient of x^{18} in
(a) $(1 + x^4 + x^7)^5$.
(b) $(1 + x^4 + x^7)^{10}$?

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.

2. A continuous function f , maps the interval $[0, 1]$ into $[0, 1]$. Show that there is a real number α in this interval, such that $f(\alpha) = \alpha$. (A diagram is not sufficient.)
3. Triangle ABC has $\angle CAD = \alpha$, $\angle CBA = 2\alpha$ and $AB = c$. Let D be the foot of the perpendicular from C to AB . Find the limiting value of AD as $\alpha \rightarrow 0$.