MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 14, August 18, 2015[1]

1. A number of cards are to be taken out of a standard 52-cards deck. If the resultant deck contains all four aces, with a probability to select all aces being $1/1001$. How many cards had been taken out of the original deck?

2. Let $n!$ denote the factorial of n; i.e $n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$. Find the largest integer n, such that $1 + 2! + 3! + \ldots + (n-1)! + n!$ is a perfect square.

3. Let $a_1, a_2, \ldots, a_100$ be a sequence of consecutive positive integers. Find the minimum value of $\sqrt{a_2 + a_3 + \ldots + a_{99}} - \sqrt{a_1 + a_{100}}$.

4. A 6-digit number is increased 6 times when its last 3 digits are carried to the beginning of the number without their order being changed. Find this number.

5. A large number of brown, green and yellow frogs are wandering around on an island. Whenever two frogs of different colours meet each other, they change immediately into two frogs of the third colour. More than two frogs never meet simultaneously. If there are initially 2014 brown frogs, 2015 green frogs and 2016 yellow frogs on the island, is it possible that at some future time all the frogs will have the same colour?

6. On the sides of triangle $\triangle ABC$, three similar triangles are constructed with $\triangle YBA$ and $\triangle ZAC$ in the exterior and $\triangle XBC$ in the interior. Above, the vertices of the triangles are ordered so that the similarities takes vertices to corresponding vertices; for example, the similarity between $\triangle YBA$ and $\triangle ZAC$ takes Y to Z, B to A and A to C.

   (a) Show that the triangles YBX and ZXC are similar to ABC.

   (b) Use part (a) or otherwise, prove that $AYXZ$ is a parallelogram.

[1]Some problems from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*.

Senior Questions

1. Express
$$\sum_{n=1}^{25} \frac{2n-1}{n(n+1)(n+2)}$$
as a fraction of two co-prime positive integers.

2. A small circle is locate inside a larger circle, with the two circles touching at the point A. P is a point on the large circle, and T is a point on the small circle such that PT is tangent to the small circle. Prove that provide $P \neq A$, the ratio of lengths of PT and PA is the same for any point P on the large circle.

3. A boy and a girl were sitting under a tree. Then twenty more children one after another came to sit under the tree, each taking a place between already setting children. Let us call a girl brave if she sat down between two boys, and let us call a boy brave if he sat down between two girls. It happened, that in the end all girls and boys were sitting in the alternating order. Is is possible to uniquely determine the number of brave children?