



## MATHEMATICS ENRICHMENT CLUB.

### Problem Sheet 12, August 2, 2015<sup>1</sup>

- Show that the number  $2017^{46} - 46$  is not prime.
  - Show that the number  $2017^{46} + 46$  is not prime.
- The polynomial  $x^3 + px^2 - x + q$  as a factor  $(x - 5)$  and a remainder of 24 when divided by  $(x - 1)$ . Find the values of  $p$  and  $q$ .
- Let  $A(-13, 14)$ ,  $B(-3, -5)$  and  $C(7, 11)$  be the coordinate of vertices of a triangle  $\triangle ABC$ . Find the coordinate of the point  $P$  inside  $\triangle ABC$  such that the triangle  $\triangle PBC$  is equilateral.
- An arithmetic sequence has positive integral entries. The sum of some 4 consecutive terms is 30. The some of some 5 consecutive terms is 30. Find the least number of terms overlapping in the two subsequences.
- Let  $f(x) = \frac{4^x}{4^x + 2}$ . Calculate

$$f\left(\frac{1}{2015}\right) + f\left(\frac{2}{2015}\right) + \dots + f\left(\frac{2014}{2015}\right).$$

- In a convex quadrilateral  $ABCD$  the diagonals are perpendicular. Points  $M$  and  $N$  are marked on the sides  $AD$  and  $CD$  respectively. Suppose the  $\angle ABN$  and  $\angle CBM$  are right-angles, prove that the lines  $AC$  and  $MN$  are parallel.

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<sup>1</sup>Some problems from UNSW's publication *Parabola* and the *Tournament of Towns in Toronto*.

### Senior Questions

1. Let  $[x]$  denote the greatest integer less than or equal to  $x$ . If  $\alpha, \beta$  and  $\gamma$  are the roots of the function  $f(x) = x^3 + x^2 - 5x - 1$ , find  $[\alpha] + [\beta] + [\gamma]$ .
2. Show that if  $n$  and  $m$  are positive integers, then  $(nm)! \geq (m!)^n (n!)^m$ .
3. Find all positive numbers  $x$  and  $y$  such that

$$x^{x+y} = y^{x+2y} \quad \text{and} \quad x^{2x+y} = y^{x+4y}.$$