Solution Sheet 6, June 4, 2012

Answers

1. 6
2. only c and d are always true.
3. $x = 170, y = 13$ and $x = 170, y = 3$
4. $5\sqrt{2}$
5. Notice that using only the rules 1 and 2 ($(2x, y)$ and $(x, 2y)$ resp.) we can obtain all points of the form $(2^n, 2^m)$ and $\gcd(2^n, 2^m) = 2^{|m-n|}$: a power of 2. Furthermore, the operations $(x - y, y)$ and $(x, y - x)$ (as used in Euclid's algorithm), preserve the gcd. Hence points with a gcd that is not a power of 2 cannot be reached.

   Conversely, these are the only points that can be reached. If $\gcd(a, b) = 2^m$, then $a = 2^m a', b = 2^n b'$ with $\gcd(a', b') = 1$. The point (a, b) can be reached from (a', b') using rules 1 and 2 (apply each m and n times resp.).

   Assume $a' < b'$. Since both a', b' are odd. $a' + b'$ is even, and can be reached from the point $(a', \frac{a'+b'}{2})$. Notice that this point is closer to $(1, 1)$ than (a', b') was.

   Continue this process until $a' = b'$, since $\gcd(a', b') = 1$, this point is $(1, 1)$.