Solution Sheet 16, September 10, 2012

Answers

1. 3 boxes of apples, 1 box of oranges, 4 boxes of lemons.

2. Find the area of the larger square in two different ways.

3. Let the rectangle have sides of length a, b where

$$a + b = 10. \tag{1}$$

By Pythagoras' theorem, the diagonal d has length

$$d^2 = a^2 + b^2 \tag{2}$$

Sub equation **??** into equation 2, the result is

$$d^2 = 2a^2 - 20a + 100$$

Use the formula for the minimum value of a parabola to find that $a = b = 5$. Hence $d = 2\sqrt{5}$.

4. $y_n = 1 \pmod 8$ but $x_2 = 3 \pmod 3$ and $x_3 = 5 \pmod 8$. For larger values of n: $x_n = 3 + 2 \times 5 \pmod 8 = 5 \pmod 8$ or $x_n = 5 + 2 \times 3 \pmod 8 = 5 \pmod 8$

5. Using $a^2 + b^2 \geq 2ab$,

$$
\begin{aligned}
2(a^2 + b^2 + c^2) &\geq 2(ab + bc + ca) \\
3(a^2 + b^2 + c^2) &\geq (a^2 + b^2 + c^2) + 2(ab + bc + ca) \\
3(a^2 + b^2 + c^2) &\geq (a + b + c)^2 \\
\frac{a^2 + b^2 + c^2}{3} &\geq \left(\frac{a + b + c}{3}\right)^2
\end{aligned}
$$