1. If \( f(n) = (n - 1)f(n - 1) \) and \( f(1) = 1 \) find \( f(4) \).

2. The product of the ages in years of two adults is 770. What is the sum of their ages?

3. (a) How many positive integers are there \( \leq 100 \) which have no factors, except 1, in common with 100?

(b) What is their sum?

4. If \( x_1 = 3 \), the recurrence \( x_{n+1} = x_n^2 - 10x_n \), gives the sequence \( 3, -21, 651, 417291 \ldots \) and the numbers increase without bound. Find all the values for \( x_1 \) so that the sequence does NOT increase without bound.

5. Solve the simultaneous equations:

\[
\begin{align*}
x + yz &= 2 \\
y + xz &= 2 \\
z + xy &= 2.
\end{align*}
\]

6. Two circles \( C_1, C_2 \) with centres \( O_1, O_2 \) are externally tangent at the point \( P \). A straight line through \( P \) meets \( C_1, C_2 \) respectively at \( A \) and \( B \). Show that the tangents to the circles at \( A \) and \( B \) are parallel.

7. Let \( ABCD \) be a trapezium with \( AB \parallel CD \). Let \( P \) be the intersection of the diagonals \( AC \) and \( BD \).

(a) Show that the triangles \( APD \) and \( BPC \) have the same area.

(b) Given that \( APB \) has area 1 cm\(^2\) and that \( APD \) has area 4 cm\(^2\), find the area of \( ABCD \).

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1Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.
Senior Questions.

1. Find $\int \frac{1}{x + \sqrt{x}} \, dx$.

2. Find $\lim_{n \to \infty} \frac{n!}{n^n}$.

3. Prove that

$$1 \times 3 \times 5 \times \ldots \times (2n - 1) = \frac{(2n)!}{2^n n!}.$$